NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1352

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INTRODUCTION

A great number of discussions have long been made to explain the cause of transition from laminar to turbulent flow in the boundary layer; none of them has as yet led to a definite conclusion. Many attempts have been put forward to calculate the transition theoretically by considering the stability of the boundary layer with superimposed infinitesimal disturbances. But the results do not agree with experiments except in the case of the transition in the flow between concentric rotating cylinders discussed by Taylor (ref. 1). (According to Nature, December 14, 1946, it is reported that Dryden has succeeded in showing the agreement between the calculated result by stability theory and the transition observed in a wind tunnel of low stream turbulence, but we know nothing about the details.)

Another example of the breakdown of laminar flow is the case when the boundary layer flow separates from the surface while it is laminar. There are two cases; in one case the separated flow leaves the surface forever, while in the other case the separated flow becomes turbulent due to its instability and reattaches to the surface as a turbulent boundary layer. The latter case is a kind of transition, and there are many experimental examples in which such a sort of transition appears. The transition which occurs when there is a steep rise in surface pressure distribution seems to be mostly due to this mechanism. The sudden change in drag or maximum lift occurring in a certain range of Reynolds numbers is considered to depend upon whether or not the separated flow reattaches to the surface. In spite of such an importance in practice, very few studies have been made concerning this problem, except for Von Doenhoff's kinematical speculation deduced from a simple experiment (ref. 2), Tani's postulate of constant Reynolds number at the laminar separation point based on dimensional analysis (ref. 3), and Inoue's discussion of Von Doenhoff's speculation (ref. 4). Experiments were therefore made to examine the effects of the geometry of body surface, Reynolds number, stream turbulence and a roughness element (wire) on the reattachment of separated flow, with a view to inquiring about the mechanism of transition caused by separation and hence to understanding the effect of Reynolds number on the flow characteristics.

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I. MEASURING APPARATUS

The wind tunnel used for the experiments was of the N.P.L. type, having a test section 50 cm × 50 cm. The upper wall of the tunnel could be opened by means of hinges so that we were able to change as well as work on the model. The upper wall had a central groove, into which a sliding cover was put so as to be able to move without air leakage in the stream direction. The cover carried a measuring instrument, pitot tube or hot-wire anemometer, and the micrometer which moved the instrument up and down. One of the side walls was made of plate glass which enabled us to observe the flow from outside.

The models used were a wooden hollow flat plate 5 cm thick, 60 cm long, 50 cm wide, and a duralumin flat plate 7 mm thick, 70 cm long, 50 cm wide, each spanning the test section of the wind tunnel. The rear part of the model, 30 cm long, was hinged to the forward part, so that the model could be bent at the hinge point. Since a sharp rise in pressure occurs at the bent corner, the separation of flow was fixed there. Thus, we could simplify the experimental conditions by eliminating the parameter concerning the point of separation. In order to vary the condition at the separation point, we varied the angles α and β of the forward and rear parts respectively with respect to the direction of wind tunnel stream (fig. 1a).

The separated flow was examined by (1) measuring the turbulence by the hot-wire anemometer, (2) observing the flow pattern as indicated by the lampblack in oil put on the surface, and (3) measuring the dynamic pressure 0.15 mm above the surface by a pitot tube in contact with the surface. The hot-wire was a platinum wire of 0.05 mm diameter, and had a time constant $M = 4.2 \times 10^{-3}$ sec. It was used in connection with a compensating circuit. The pitot tube was made from copper tubing of 0.5 mm outside diameter, flattened to a mouth of 0.1 mm inner diameter and 0.1 mm thickness. The pressure distribution along the surface was measured by a static-pressure tube made from copper tubing of 2 mm outside diameter, the end of which was closed and in which 4 holes of 0.2 mm diameter were drilled around the tube.

II. GENERAL OBSERVATION BY THREE METHODS OF MEASUREMENTS

(A) Comparison Between the Pitot Tube and Lampblack Measurements

The measurements were made on a duralumin plate with $\alpha = -9^{\circ}30'$ and $\beta = 7^{\circ}$ for two wind speeds, 8.1 m/sec and 15.3 m/sec.

The dynamic pressure measured by the pitot tube in contact with the surface is shown in figure 1d. The dynamic pressure is zero at the bent corner, for example, the separation point, becomes negative, then increases suddenly, reaches a maximum, and finally decreases gradually. The lampblack method enables us to observe, as shown in figure 1f, the region of lampblack at rest, the region of lampblack accumulation, and the starting line of reverse flow. When the Reynolds number increases, the region of lampblack at rest disappears, and, at the same time, both the region of lampblack accumulation and the starting line of reverse flow move upstream. Generally speaking, the downstream edge of lampblack accumulation coincides with the position where the dynamic pressure begins to increase, and the starting line of reverse flow coincides with the maximum point of the dynamic pressure. It seems therefore that a thin elongated vortex extends downstream of the separation point when the separated flow reattaches to the surface as a turbulent boundary layer.

The pressure distribution along the surface is shown in figure 1e. As the Reynolds number increases, the extent of the separation region decreases accompanied by a considerable change in the pressure distribution; thus the pressure coefficient at the bent corner is almost doubled and the resistance halved. It will be seen therefore that there is a considerable effect of Reynolds number due to the change of the extent of the transient region, although the position of separation is fixed. We expect to consider this problem in more detail at another time.

There are cases where the dynamic pressure indicated by the pitot tube does not decrease but sometimes increases gradually after reaching a maximum value. In such cases, the reattachment of the separated flow is found to be incomplete, now attaching and now leaving the surface in turn.

(B) Observation of Turbulence by a Hot-Wire Anemometer

The measurements were made on a wooden plate with $\alpha = -5^{\circ}$ and $\beta = 13^{\circ}$ for wind speeds U_s at the separation point from 14.6 m/sec to 16 m/sec. Moving the hot-wire downstream keeping the distance at about 0.5 mm from the surface, we observed just downstream of the separation point velocity fluctuations of low frequency, the amplitude increasing but gradually. At the same time, high frequency fluctuations occurred, the amplitude being still small. Finally, violent high frequency fluctuations appeared. Downstream of the point of reattachment, the frequency became lower and the amplitude decreased.

When the hot-wire was moved away from the surface, the fluctuations became smaller and smaller, until they were equal to those of the general stream. The boundary of this region looks similar to the upper boundary of turbulence diffusion as speculated by Von Doenhoff (fig. 1b).

The description mentioned previously is quite similar to that hitherto observed in the usual transition of the boundary layer. We are therefore led to the suggestion that most of the transition occurs by means of such a mechanism. According to Von Doenhoff's speculation, the triangular region ABC of figure 1b is considered to be quiet, while according to our observations (A) and (B), a vortex seems to exist here, thus causing violent velocity fluctuations. We will consider this point again in section VI.

When the flow leaves the surface without reattachment, the circumstances are different from those described previously, the velocity fluctuations decreasing as we approach the surface and increasing as we proceed downstream.

III. VARIATION OF THE REATTACHMENT WITH α AND β

Measurements were made on a duralumin plate by the pitot tube method to determine the variation of the point of reattachment for various values of α, β, and wind speed. The wind speed U_s, the speed outside the boundary layer at the separation point, was varied from 7 m/sec to 15 m/sec. The results are summarized in table 1.

If we construct a Reynolds number from U_s and L; $R_L = \frac{LU_s}{\nu}$, where L is the distance between the bent corner and the point of reattachment, we find that the values are approximately constant independent of the wind speed for a fixed set of values of α and β. Plotting the values of R_L on the corresponding geometrical configurations, we find the points lying approximately on a straight line for a fixed value of α.

This result reminds us of Von Doenhoff's hypothesis, but it is difficult in our case to define the direction of separated flow, because there is no tangent line at the bent corner. Most naturally, we first assume the direction to be the bisector of the angle formed by the downstream surface and the elongation of the upstream surface. This bisector and the straight line through the reattachment points makes an angle γ equal to $13^\circ 40'$ independent of the value of α, but the distance from the separation point to the intersection of the two lines gives the Reynolds numbers R_x = 17,200, 13,800, and 12,300 for $\alpha = -9^\circ 30'$, $-12^\circ 30'$, and $-14^\circ 30'$, respectively.

We then assume that the direction of separated flow coincides with that of the general stream for the case of $\alpha = -9^\circ 30'$, and that it increases by an amount $\frac{1}{2}\Delta\alpha$, where $\Delta\alpha = -9^\circ 30' - \alpha$. The distances R_x

and angles γ defined similarly as in the foregoing paragraph then become R_x = 26,000, 25,000, 23,700 and $\gamma = 17^\circ 30'$, $17^\circ 30'$, $15^\circ 50'$ for $\alpha = -9^\circ 30'$, $-12^\circ 30'$, $-14^\circ 30'$, respectively. Averaging the results, we have R_x = 25,000 and $\gamma = 17^\circ$ (fig. 2a).

If the angle β is increased such that the Reynolds number R_L exceeds 75,000, the reattachment becomes incomplete and, as already mentioned, the dynamic pressure does not decrease after reaching a maximum. Such values of R_L, which are marked by parentheses in table 1, do not lie on the straight line. Even when β is below the limiting value, but close to it, a similar tendency appears when the wind speed is low.

The velocity profile of the boundary layer at the separation point was measured for the case $\alpha = -9^\circ 30'$, $\beta = 7^\circ$. The values of Reynolds number R_θ based on the momentum thickness were found to be from 178 to 249 when the separated flow reattaches to the surface completely. So Tani's postulate on the critical value of R_θ appears to have to be reconsidered in our case.

IV. EFFECT OF STREAM TURBULENCE

In order to see the effect of stream turbulence on the reattachment of laminar separation, measurements were also made with a square mesh grid (composed of wooden round bars of 6.35 mm diameter at a mesh length 25.4 mm) placed at a distance 68.4 cm upstream, or with a wire screen (composed of 1 mm diameter wire at a mesh length 5 mm) placed at 68 cm upstream of the leading edge of the model. The values of turbulence intensity at the separation point were 1.7 percent and 1.0 percent, respectively, while the value for the bare tunnel was 0.15 percent. The results are also summarized in table 1.

Analyzing the results by the first method, in which the direction of separated flow is assumed to be the bisector of the angle, we have R_x = 13,800, $\gamma = 15^\circ$ for the case of wire screen, and R_x = 17,200, $\gamma = 15^\circ 40'$ for the case of grid. Analyzing by the second method, in which the direction of separation is assumed to coincide with the general stream for the case $\alpha = -9^\circ 30'$, and to increase by an amount $\frac{1}{2}\Delta\alpha$ for the other cases, we have R_x = 25,000, $\gamma = 20^\circ 30'$ and R_x = 26,000, $\gamma = 22^\circ 50'$ for the cases of wire screen and grid, respectively. It seems not unreasonable to average the results to get R_x = 25,000 and $\gamma = 21^\circ 50'$. This supports the second method of analysis in that the final result becomes simpler. The values are shown in figure 2b.

Similarly to the previous result, when the value of R_L exceeds a certain critical value, the reattachment becomes incomplete, the dynamic pressure showing no decrease after reaching a maximum, and the value of R_L deviates from the straight line. The critical value of R_L seems to lie in the range from 48,000 to 56,000 both for the cases of turbulence intensity 1.0 percent and 1.7 percent, although the measurements were insufficient to determine the exact value.

V. THE EFFECT OF A PIANO WIRE PLACED ABOVE THE SEPARATION POINT

Measurements were made to examine the effect of a piano wire of 0.5 mm diameter, which was placed at various distances, $y = 2.04$, 3.0, 4.09, 5.0, and 7.0 mm, above the bent corner. The angle β was varied from 6° to 9° and the wind speed from 8 m/sec to 16 m/sec.

Two kinds of Reynolds number may be considered as the parameter representing the effect of the wire. The one is the Reynolds number R_d based on the diameter d of the wire and the wind speed U at that position, and the other is the Reynolds number R_y based on the height y of the wire above the surface and the wind speed U. Since the thickness of the boundary layer was from 2.2 mm to 2.4 mm, the value of U must be smaller than U_s when $y = 2.04$ mm. When y is large, the value of U is again expected to become smaller than U_s due to the character of potential flow. However, we simply adopt U_s in place of U, because we made no measurement.

The relation between R_L and R_y is shown in figure 3 for different values of β. The measured points lie nearly on a single curve, there being a tendency, however, such that R_L is larger when R_d is larger. This tendency seems to be associated with the fact that the drag coefficient of the circular cylinder decreases from 1.27 to 1.13 in the range of R_d from 250 to 530, which correspond to the present measurement. Determining the faired values of R_L from the curves of figure 3 for various values of R_y and β (table 2), and plotting R_L on the corresponding configurations as shown in figure 2c, the straight lines drawn through the points, except for $\beta = 8^\circ$, seem to converge to a point which is $R_x = 25{,}000$ distance from the corner in the direction of the general stream rotated by an angle $\frac{1}{2}\Delta\alpha = 1^\circ30'$. The values of angle γ made by each straight line with the reference line are given in table 2. It seems somewhat strange that R_x remains constant even if γ exceeds 90°. But it is to be noted that the experimental results cannot be explained if we assume the turbulence to diffuse right from the position where the wire is placed.

VI. DISCUSSION OF RESULTS AND CONCLUSION

We find from the preceding results that the reattachment of the separated flow may be simply interpreted in terms of R_x and γ as put forward by Von Doenhoff, and that a reverse-flow vortex exists and hence the velocity fluctuation is most violent in the region which is considered to be quiet according to Von Doenhoff's hypothesis. These two facts apparently contradict each other, but they may be reconciled if we consider that Von Doenhoff's hypothesis is responsible for explaining the transient process from the initial separation of laminar flow to the finally arrived stationary state in which the flow reattaches to the surface, and that the reverse-flow vortex plays an important role in the stationary state. In other words, the separated flow becomes unstable and produces turbulence, which extends to the surface and assists the separated flow in reattaching to the surface. During this process, the fluid in the triangular region remains at rest, but it develops into a vortex due to the dragging action of the outside flow. The vortex then acts positively to attract the separated flow toward the surface. Since the vortex maintains itself by the circulation continuously supplied from the separation of the boundary layer, there must be a balance between the strength of the vortex and the circulation at the separation point. If the Reynolds number R_L becomes too large, which is proportional to the length of the vortex, it becomes impossible to supply circulation sufficient to maintain the vortex against the dissipation. Then the vortex becomes unstable and makes the reattachment incomplete. This explains the existence of the upper limit of the value of R_L. It seems at first difficult to understand the fact that R_x is independent of the stream turbulence, but such a circumstance may be suggested from the researches made by Schiller and his coworkers (ref. 5), which are briefly as follows: For flow in pipes and channels, there is a lowest critical Reynolds number such that for Reynolds numbers less than the critical value all disturbances, however great, are damped out sufficiently downstream. The critical value is 1160, when the radius a of the pipe, the half width a of the channel, or the thickness δ of the boundary layer is used as a reference length in defining the Reynolds number. Schiller suggests that a vortex street arising from the edge of the entry develops into turbulence when Z/ν exceeds 1170, where Z is the circulation of the vortices contained in the length equal to a or δ. It will be seen therefore that the lowest value of the Reynolds number of the vortex street coincides roughly with the lowest critical Reynolds number of the flow, and that the vortex street always develops if $Z/\nu > 1170$ and $\delta U/\nu > 1160$, but dies away if one of the inequalities does not hold. This fact seems to be essentially the same as our finding that R_x is constant. It will be most natural in our case to take Z equal to $u_1\delta$, where u_1 is the velocity outside the boundary layer and δ is the thickness of the boundary layer; Z/ν then becomes the Reynolds number of the boundary layer. If Z/ν is less than 1170, then the separated flow is stable so that no vortex is produced, and therefore insufficient circulation is supplied for maintaining the reverse-flow vortex even if it were produced. This is considered to lead to the incomplete reattachment of the separated flow. We observed several examples in which the dynamic pressure indicated by the pitot tube in contact with the surface is constant after reaching a maximum value, when R_L is comparatively high and the wind speed is low. Moreover, according to the additional measurements for the case $\alpha = -9^\circ 30'$, $\beta = 11^\circ$, which is very near to the critical value of R_L, the flow was found to be stable for U_s higher than 10.35 m/sec, but unstable for the range from 8 m/sec to 3 m/sec. The Reynolds numbers $\delta U_s/\nu$ corresponding to the unstable cases are from 1326 to 818, thus agreeing with the fact mentioned previously.

The criterion due to Tani is formally the same, but the value given by him is too high. Moreover, this is the necessary condition, but not the sufficient condition, so that the condition alone cannot determine what happens.

Summarizing the results of our study, we have as follows:

1. Von Doenhoff's speculation is valid for explaining the transient process during which the initial separation of laminar flow develops into a stationary state with the reattachment of flow. But in the stationary state, a reverse-flow vortex produced downstream of the separation point attracts the separated flow toward the surface.

2. The stability of the separated laminar flow breaks down when the Reynolds number R_x exceeds 25,000, this critical value being independent of the stream turbulence. But the instability does not appear when the Reynolds number based on the boundary layer thickness at separation is less than 1200.

3. After the breakdown of laminar flow, the turbulence spreads at a certain angle which increases as the stream turbulence increases.

4. The reattachment of the separated flow becomes incomplete when the Reynolds number R_L exceeds a certain critical value, which seems to decrease as the stream turbulence increases.

5. Even if the separation point is fixed, the extent of the separation region affects considerably the pressure distribution as a whole as well as the characteristics of the reattached boundary layer. Therefore, the experimental data on a turbulent boundary layer whose condition of transition is not clear seem to be insignificant.

VII. NOTE ON TRANSITION FOR SPHERES AND CIRCULAR CYLINDERS

According to Von Doenhoff, the critical Reynolds number R_c where a sudden decrease in drag of spheres or circular cylinders begins is given by

$$R_c = R_x \frac{U_0}{U_s} \frac{2}{\tan \frac{\gamma}{2}}$$

where U_0 is the speed of undisturbed stream. R_x is independent of the stream turbulence as mentioned previously. U_0/U_s is also nearly independent of the stream turbulence. Hence, assuming R_c to be proportional to the conventional critical Reynolds number of the sphere, we get for the so-called turbulence factor (T.F.) the expression

$$\text{T.F.} = \frac{R_{c0}}{R_c} = \frac{\tan \frac{\gamma}{2}}{\tan \frac{\gamma_0}{2}} \approx \frac{\gamma}{\gamma_0}$$

where the subscript 0 refers to the condition of the free atmosphere. Assuming the relation between the turbulence factor and the percentage intensity of turbulence in the form $\text{T.F.} - 1 = 84(u'/U)$, and adjusting the constant so as to make $\gamma = 17^\circ$ for $u'/U = 0.15$ percent, we have

$$\gamma = 15.1^\circ \times \text{T.F.} = 15.1^\circ + 1267^\circ \frac{u'}{U}$$

If we estimate γ by this formula for the case $u'/U = 1.0$ and 1.7 percent, we have $\gamma = 27.8^\circ$ and 36.7°, respectively, which, however, are far greater than the experimental values mentioned above. This suggests the importance of taking account of the effect of surface curvature distribution. It seems therefore necessary to make extensive measurements on the effect of surface curvature distribution, before we are in a position to discuss the effect of Reynolds number and, especially, the effect of stream turbulence (concept of effective Reynolds number) on the characteristics of flow.

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REFERENCES

1. Taylor, G. I.: Phil. Trans. Roy. Soc., A, vol. 223, 1923, p. 289.

2. Von Doenhoff, A. E.: Proc. Fifth Int. Congr. App. Mech., 1938, p. 311.

3. Tani, I.: J. Soc. Aero. Sci. Japan, vol. 7, 1940, p. 229.

4. Inoue, E.: J. Soc. Aero. Sci. Japan, vol. 10, 1943, p. 107.

5. Schiller, L.: Proc. Fifth Int. Congr. App. Mech., 1938, p. 315.

TABLE 1

α	β	$\frac{\alpha + \beta}{2}$	$\frac{\beta - \alpha}{2}$	$\beta + \frac{\Delta\alpha}{2}$	R_L	R_L with grid or screen
-9°30'	7°	-1°15'	8°15'	7°	41500	grid 36500
	7°55'	-0°47.5'	8°42.5'	7°55'	47500	39000
	9°10'	-0°10'	9°20'	9°10'	54500	43000
	10°30'	0°30'	10°	10°30'	62800	47000
	11°45'	1°7.5'	10°37.5'	11°45'	(74800)	(56000)
-12°30' $\left(\frac{\Delta\alpha}{2} = 1°30'\right)$	6°	-3°15'	9°15'	7°30'	43500	screen 39000
	7°	-2°45'	9°45'	8°30'	48500	42000
	8°	-2°15'	10°15'	9°30'	54000	46000
	9°	-1°45'	10°45'	10°30'	63500	(56000)
-14°30' $\left(\frac{\Delta\alpha}{2} = 2°30'\right)$	4°	-5°15'	9°15'	6°30'	40500	
	5°	-4°45'	9°45'	7°30'	44500	
	6°	-4°15'	10°15'	8°30'	52000	
	7°	-3°45'	10°45'	9°30'	59700	
	8°	-3°15'	11°15'	10°30'	74000	
	9°	-2°45'	11°45'	11°30'	(84700)	
	10°	-2°15'	12°15'	12°30'	(113000)	

TABLE 2

	R_y	1500	2000	2500	3000	4000	5000	6000	7000
$R_L \times 10^{-4}$	β = 6°	2.20	2.58	2.92	3.19	3.60	3.85	4.04	4.20
	7°	2.15	2.60	3.00	3.30	3.76	4.10	4.34	4.50
	8°	2.01	2.46	2.90	3.29	3.93	4.41	4.70	4.81
	9°	2.12	2.69	3.23	3.70	4.51	5.10	5.44	5.50
γ		138.5°	78°	43.5°	31.5°	24.5°	21.5°	20°	19°

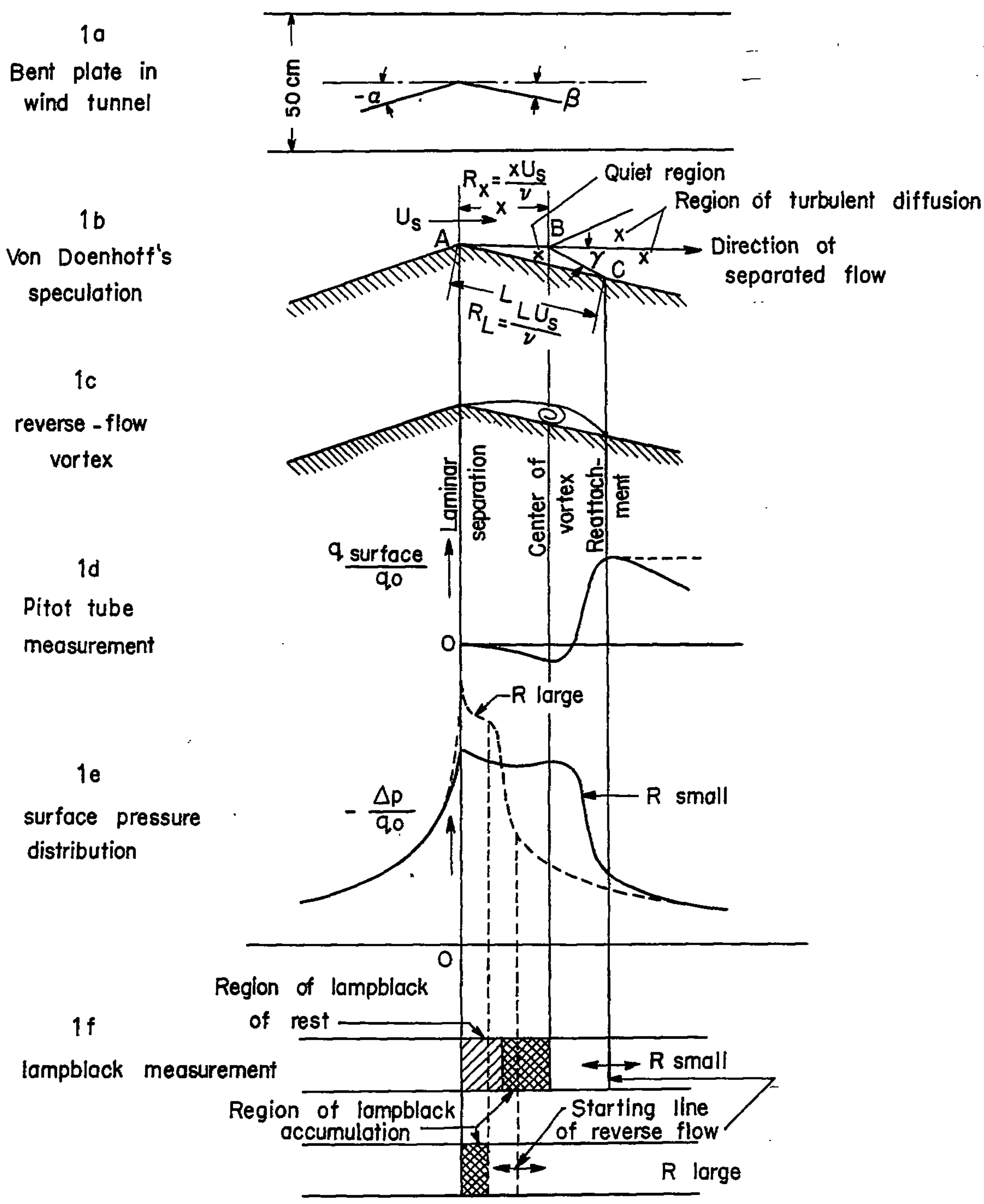
1a
Bent plate in wind tunnel
50 cm
-α
β
1b
Von Doenhoff's speculation
$R_x = \frac{xU_s}{\nu}$
Quiet region
Region of turbulent diffusion
U_s
A
B
C
x
γ
Direction of separated flow
L
$R_L = \frac{LU_s}{\nu}$
1c
reverse-flow vortex
Laminar separation
Center of vortex
Reattachment
1d
Pitot tube measurement
$\frac{q \text{ surface}}{q_o}$
0
R large
1e
surface pressure distribution
$-\frac{\Delta p}{q_o}$
R small
0
Region of lampblack of rest
1f
lampblack measurement
R small
Region of lampblack accumulation
Starting line of reverse flow
R large

Figure 1.

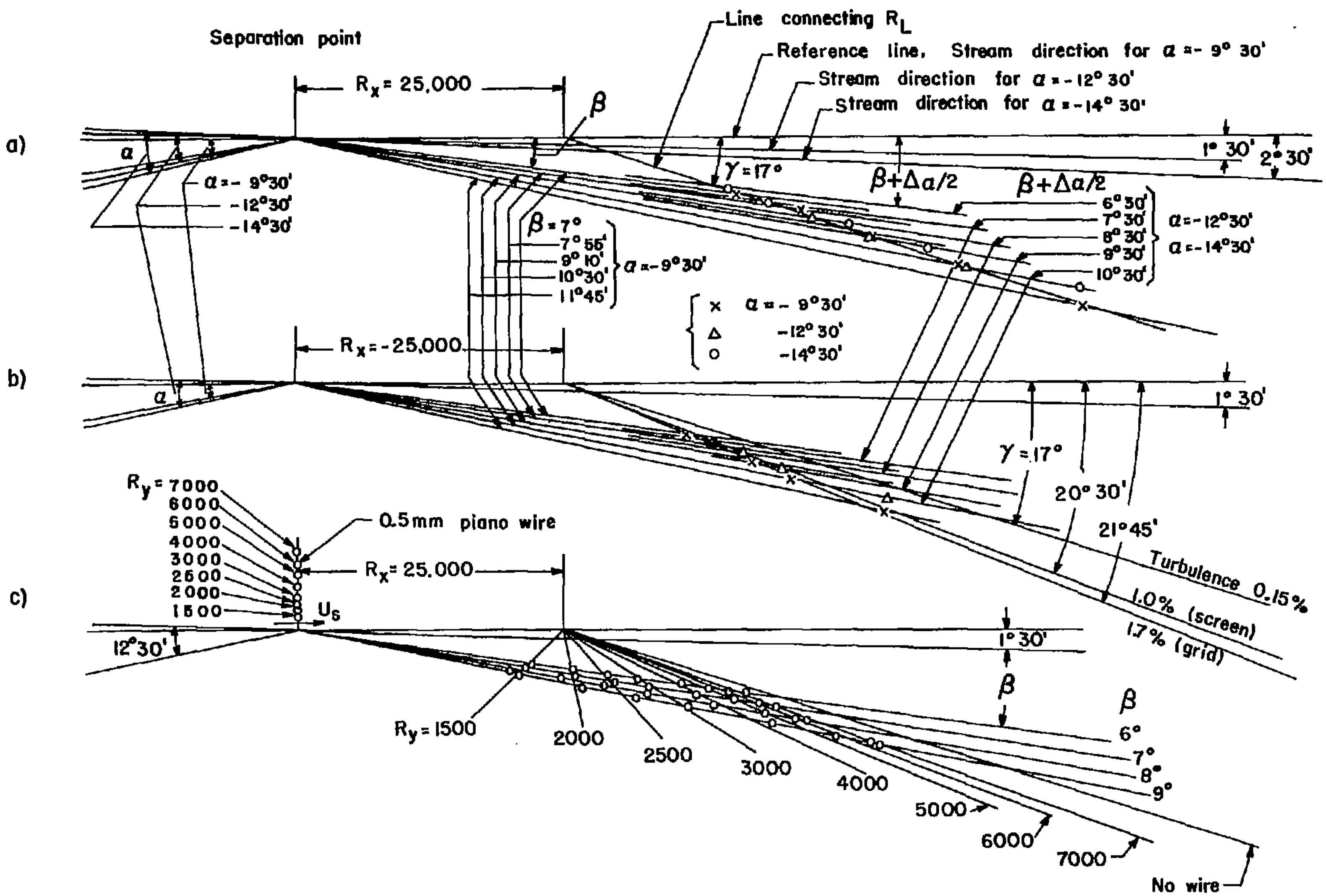

Figure 2.- Effects of angles α and β, stream turbulence, and piano wire on the position of reattachment of separated flow.

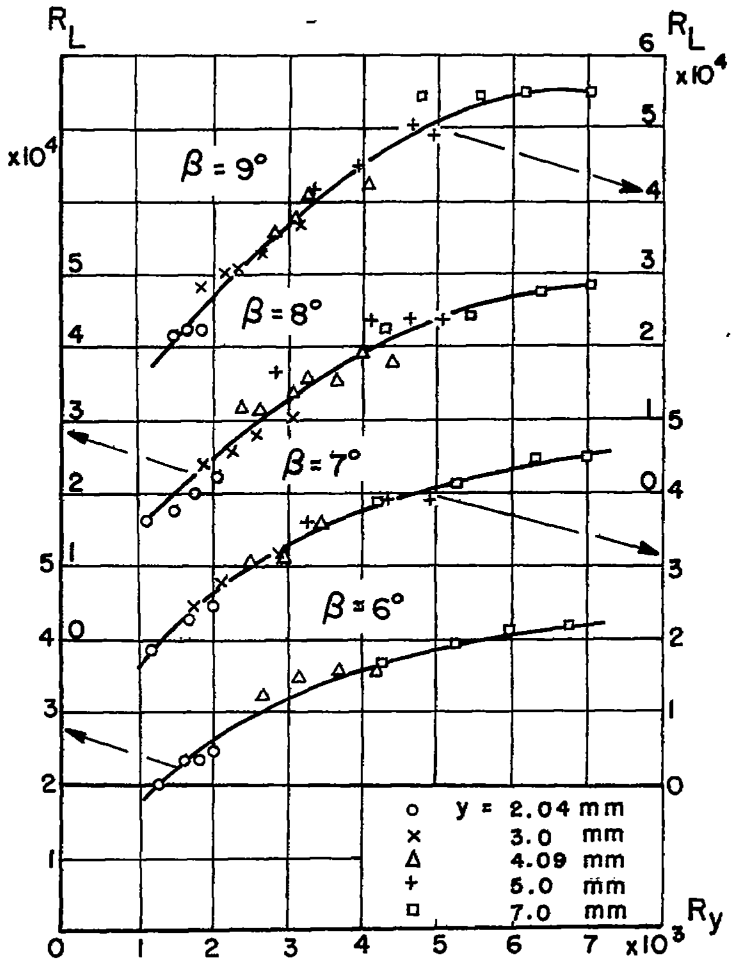

Figure 3.- Effect of a piano wire on the position of reattachment of separated flow.